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# Coping with the backorder in the loss-averse newsvendor model

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**Abstract.** We introduce loss aversion into the newsvendor model with a backorder. By introducing the backorder rate $w$ and the loss aversion coefficient $\lambda$, we propose a novel utility function for the newsvendor. Firstly, we find that the optimal order quantity by maxmizing the expected utility for the loss-averse newsvendor permitted backorder who is risk-neutral is smaller than the newsvendor without backorder. For another case, if the loss-averse newsvendor is risk-averse, we obtain the optimal order quantity by maximizing CVaR about utility to reduce the risk that originates from the fluctuation in the market demand. It is found that the optimal order quantity is decreasing with the confidence level $\alpha$, and is decreasing with the backorder rate $w$, too. We also found that the optimal order quantity is smaller than the expected utility under this optimal order quantity. Further, a numerical example is given to illustrate the obtained results. Finally, some management insights are suggested for the loss-averse newsvendor model with a backorder.

**Key words.** Newsvendor model, conditional value-at-risk (CVaR), optimal order; expected utility function, backorder rate, confidence level.

## 1. Introduction

In the real world, decision can be found in various areas, such as organizational behaviour, financial markets, supply chain management and labor supply. Therefore, the newsvendor model is proposed and the application in production plan of the model is successful [1-3]. In the classical newsvendor model, the studies are focused on ways to obtain the optimal order quantity with different decision criterions and/or preferences to maximize the expected utility for the newsvendor. However, some studies found that the realized order quantity of the manager in practice always deviates from the expected profit maximization order quantity. Then, the "decision

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bias" is proposed to the decision framework of the classical newsvendor model to explain this phenomenon [4-6]. For example, J. Sun and X. Xu introduced the loss aversion into the decision framework and obtained the optimal order quantity to maximize the expected utility and to maximize CVaR about utility for the loss-averse newsvendor who is risk-averse, respectively. In the study about the loss aversion in the newsvendor model, it is found that the more loss-averse the newsvendor is, the less products he/she orders. Therefore, the orders may be lower the market demand quantity. In the real world, if the decision makers find that the order quantity is lower the demand quantity, they will order again to fulfil the customers who are willing to wait the new order.

The rest of this paper is organized as follows. In the following section, we give a detailed description on the problem studied in this paper. Section 3 studies the optimal order quantity decisions for the risk-neutral newsvendor with a backorder to maximize his/her expected utility and the risk-averse newsvendor with a backorder to maximize his/her CVaR about expected utility respectively, the properties of the two optimal order quantities are presented as well. Section 4 gives a numerical example and sensitivity analysis to verify the obtained results in this paper, and some management insights for the newsvendor model are suggested by the numerical results, with the conclusions given in Section 5.

## 2. Presentation and motivation

For the newsvendor problem with a backorder case, suppose the market demand $\zeta$ is a random variable, and its probability density function and cumulative distribution function are $f(\cdot)$ and $F(\cdot)$ respectively. In this paper, it is supposed that $F(0)=0$, $F(+\infty)=1$, $F(\cdot)$ is continuously differentiable and increasing, and the inverse of $F(\cdot)$ exists. Then, for an order quantity $q$ of the newsvendor and the realized value $D$ of $\zeta$, on the one hand, the profit realized in the selling season for the newsvendor in a backorder case can be given as$P(q)=(p\text{-}c)\min\{q,D\}+(p\text{-}c)w(D\text{-}q)^+$, (1)where $X^+=\max\{X,0\}$. Here, $p$ is the retail price of unit product, $c$ is the wholesale price of unit product from the supplier and $w$ is the backorder rate of the excess demand. In the right hand of (1), the first item $(p\text{-}c)\min\{q,D\}$ represents the profit of the newsvendor from the ordered products, the second $(p\text{-}c)w(D\text{-}q)^+$ the profit of the newsvendor from the backlogged products; on the other hand, the loss from the excess order or the shortage penalty when the selling season is due is given as$L(q)=(c\text{-}r)(q\text{-}D)^+$. (2)Here, $r$ is the salvage price of unit product which can't be sold. Without loss of generality, it is assumed that $p \geq c \geq r \geq 0$ holds. In the right hand of (2), the item $(c\text{-}r)(q\text{-}D)^+$ represents the loss of the newsvendor from excess order. Then, for the realized market demand $D$, by (1) and (2), the utility of the newsvendor to select an order quantity $q$ is given as $\Pi(q)=P(q)\text{-}\lambda L(q)=(p\text{-}c)\min\{q,D\}+(p\text{-}c)w(D\text{-}q)^+\text{-}\lambda[(c\text{-}r)(q\text{-}D)^+]$, (3)where $\lambda \geq 1$ is the loss aversion coefficient.

In the following, we focus on finding the solutions for the newsvendor with different risk references to maximize different objectives about the above utility function $\Pi(q)$.

## 3. Maxmizing the objectives of Utility $\Pi(q)$ in the newsvendor model

In this section, we will give the optimal order quantity decisions for the newsvendors to optimizing different objectives about the utility function $\Pi(q)$ introduced in section 2.

### *3.1. Optimal order quantity in maximizing expected utility function*

In this subsection, for the conventional approach to analyse the loss-averse newsvendor model with a backorder which is based on assuming that the newsvendor is risk-neutral, we will discuss the optimal order quantity decision to maximize the expected utility in formula (3).

**Theorem 3.1** For the newsvendor model with a backorder, the optimal order quantity for a risk-neutral newsvendor to maximize his/her expected utility function $E[\Pi(q)]$ ($E$ is the expectation operator) is given by

$$q^* = F^{-1}\left[\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right].$$

**Proof:** For a given order quantity $q$ of the newsvendor and a realized market demand $D$, it follows from (3) that

$\Pi(q)$ =($p$-$c$)min$\{q,D\}$+($p$-$c$)$w$($D$-$q$)$^+$-$\lambda$[($c$-$r$)($q$-$D$)$^+$]. (4)Then, it follows from $min\{q,D\}$=$q$-($q$-$D$)$^+$ and ($D$-$q$)$^+$=($D$-$q$)+($q$-$D$)$^+$ that

$\Pi(q)$ =(1-$w$)($p$-$c$)$q$+$w$($p$-$c$)$D$-[(1-$w$)($p$-$c$)+$\lambda$($c$-$r$)] ($q$-$D$)$^+$ . (5)Then, the expectation of $\Pi(q)$ is given by

$E[U(q)] = (1-w)(p-c)q+w(p-c)E(\xi)-[(1-w)(p-c)+\lambda(c-r)]\int_0^q(q-t)dF(t)$,(6) which implies$\frac{\partial E[\Pi(q)]}{\partial q} = -[(1-w)(p-c)+\lambda(c-r)]F(q)$. which implies $E[\Pi(q)]$ is concave in $q$.Then, it follows that it follows from $\frac{\partial E[\Pi(q)]}{\partial q}$=0 that $E[\Pi(q)]$ attains the maximum in

$$q^* = F^{-1}\left[\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right].$$

By this result, the optimal order quantity for a risk-neutral newsvendor to maximize his/her expected utility function $E[\Pi(q)]$ is decided by the retail price $p$, the salvage price $r$, the wholesale price $c$, the backorder rate $w$ and the loss aversion coefficient$\lambda$. Specially, if it satisfies $w$=0, which implies the newsvendor turns to be loss-averse and there is no backorder for the excess demand, then it follows from Theorem 3.1 that

$$q^* = F^{-1}\left[\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right] = F^{-1}\left[\frac{p-c}{p-c+\lambda(c-r)}\right],$$

which is same to the result obtained in Sun et al. (2015). Moreover, since it satisfies

$$\frac{p-c}{p-c+\lambda(c-r)}-\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}$$

$$=\frac{w(p-c)(c-r)}{[p-c+\lambda(c-r)][(1-w)(p-c)+\lambda(c-r)]}\geq 0,$$

which implies

$$\frac{p-c}{p-c+\lambda(c-r)}\geq\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}.$$

Then, it follows

$$F^{-1}\left[\frac{p-c}{p-c+\lambda(c-r)}\right]\geq F^{-1}\left[\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right].$$

That is to say, the optimal order quantity for a loss-averse newsvendor in a backorder case to maximize his/her expected utility function $E[\Pi(q)]$ is smaller than the optimal order quantity for a loss-averse newsvendor without backorder. This implies that if the backorder is permitted, the loss-averse newsvendor chooses a lower order quantity to reduce/control the potential risk which originates from the fluctuation in the market demand. Besides, by Theorem 3.1, the following results are obvious.

**Corollary 3.1** For the newsvendor model with a backorder, the optimal order quantity $q^*$ for a risk-neutral newsvendor to maximize his/her expected utility $E[\Pi(q)]$ is increasing in the retail price $p$, the salvage price $r$ and decreasing in the wholesale price $c$ respectively.

**Corollary 3.2** For the newsvendor model with a backorder, the optimal order quantity $q^*$ for a risk-neutral newsvendor in a backorder case to maximize his/her expected utility $E[\Pi(q)]$ is decreasing in the backorder rate $w$.

**Proof:** By Theorem 3.1, the optimal order quantity $q^*$ for a risk-neutral newsvendor to maximize his/her expected utility $E[\Pi(q)]$ is given as

$$q^*=F^{-1}\left[\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right].$$

It follows

$$\frac{\partial q^*}{\partial w}=\frac{1}{f\left[F^{-1}\left(\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right)\right]}\frac{\lambda(p-c)(c-r)}{[(1-w)(p-c)+\lambda(c-r)]^2}\leq 0,$$

which implies $q^*$ is decreasing in the backorder rate $w$. This completes the proof.

It is obvious that this result shows holds, and the loss-averse newsvendor with a backorder will not order many products before the selling season. If the order quantity is lower the market demand quantity, then he/she order the product and sell to the customs who are willing to wait for them. Of course, some customers alienated. Therefore, the backorder rate $w$ is lower than 1, but bigger than 0. For different customers, it may have different value. For different products, it may change, too. Particularly, let $w$=1, it follows that $q^*$=0. It means that if the product is lower the demand, the customers are willing to wait the newsvendor's new order instead of choosing another newsvendor or another product. That is to say, there is no customer away, even though the newsvendor has no product. Therefore, the loss-averse newsvendor with a backorder will not order any product before the selling season.

### *3.2. Optimal order quantity in maxmizing CVaR about utility*

In the above subsection, we derive the optimal order quantity for the risk-neutral newsvendor to maxmize his/her expected utility of $E[\Pi(q)]$. However, this measure ignores the risk comes from the fluctuation in the market demand and thus is insufficient for some newsvendors. In recent years, some unpredictable disasters (for example, earthquake and economic crisis) have disrupted the supply chain operations repeatedly and this leads the agents in supply chains become more sensitive to the loss variations and becomes more risk-averse. Then, many researchers pay attention to the risk analysis and risk control in the supply chains, as well as the newsvendor model .

For the order quantity $q$ of the newsvendor and the utility$\Pi(q)$ from this order quantity, we define the VaR about $\Pi(q)$ for the newsvendor as follows: $\mathrm{VaR}_\alpha[\Pi(q)]=\sup\{y \in R|\Pr\{\Pi(q)\geq y\}\geq \alpha\}$,(8)which represents the maximum utility of the newsvendor under the confidence level $\alpha$. Then, the CVaR about maximum utility $\Pi(q)$ for the newsvendor is given as $\mathrm{CVaR}_\alpha[\Pi(q)]=E[\Pi(q)|\Pi(q) \geq \mathrm{VaR}_\alpha[\Pi(q)]]$, (9)which represents the expected value of the maximum utility that exceeds the above quantile $\mathrm{VaR}_\alpha[\Pi(q)]$. By maximizing this CVaR objective, we can obtain an optimal order quantity to maximize the expected value of the maximum utility that exceeds the quantile $\mathrm{VaR}_\alpha[\Pi(q)]$, and this is just what a risk-averse newsvendor expects. Then, we have the following results about this CVaR objective.

**Theorem 3.2** For the newsvendor model, the optimal order quantity for a risk-avers newsvendor to maximize his/her CVaR about $\Pi(q)$ is given by

$$q^*_\alpha = F^{-1}\left[\frac{(1-\alpha)(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right].$$

**Proof:** See the Appendix.

Here, it is easily checked that, if it satisfies $\alpha$=0, which implies the risk-averse newsvendor turns to be risk-neutral, then it follows from Theorem 3.2 that $q^*_\alpha = q^*$ holds. Similar to Corollary 3.1, we have the following results about the optimal order quantity $q^*_\alpha$.

**Corollary 3.3** For the newsvendor model, the optimal order quantity $q^*_\alpha$ for a risk-averse newsvendor to maximize his/her CVaR about utility $\Pi(q)$ is increasing in the retail price $p$ and the salvage price $r$, and decreasing in the wholesale price $c$ respectively.

**Corollary 3.4** For the newsvendor model, the optimal order quantity $q_\alpha^*$ for a risk-averse newsvendor to maximize his/her CVaR about utility$\Pi(q)$ is decreasing in the backorder rate $w$.

The backorder rate $w$ reflects the degree of the customers willing to wait the newsvendor's new order and the bigger the backorder rate becomes, the loss-averse newsvendor with risk-averse will order less products before the selling reason to reduce the risk originating from the fluctuation in the market demand and lead to a less utility.

**Corollary 3.5** For the newsvendor model, the optimal order quantity $q_\alpha^*$ for a risk-averse newsvendor to maximize his/her CVaR about utility $\Pi(q)$ is decreasing in the loss aversion coefficient $\lambda$.

**Corollary 3.6** For the newsvendor model, the optimal order quantity $q_\alpha^*$ for a risk-averse newsvendor to maximize his/her CVaR about utility $\Pi(q)$ is decreasing in the confidence level $\alpha$.

**Corollary 3.7** For $\alpha \in [0,1]$, $E[U(q_\alpha^*)]$ is decreasing in the confidence level $\alpha$.

**Proof:** By (6), we have

$$E[U(q)] = (1-w)(p-c)q + w(p-c)E(\xi) - [(1-w)(p-c) + \lambda(c-r)] \int_0^q (q-t)dF(t),$$

It follows $\frac{\partial E[\Pi(q_\alpha^*)]}{\partial \alpha} = [(1-w)(p-c) - ((1-w)(p-c) + \lambda(c-r))F(q_\alpha^*)] \frac{\partial q_\alpha^*}{\partial \alpha}$ . (10)Since it satisfies $q_\alpha^* \leq q^*$ and $q^* = F^{-1}\left[\frac{(1-w)(p-c)}{(1-w)(p-c)+\lambda(c-r)}\right]$ , it follows that $(1-w)-((1-w)+\lambda(c-r))F(q_\alpha^*) \geq (1-w) - ((1-w)+\lambda(c-r))F(q^*) = 0$.(11) It follows from (10),(11) and Corollary 3.6 that

$$\frac{\partial E[\Pi(q_\alpha^*)]}{\partial \alpha} \leq 0,$$

which proves $E[U(q_\alpha^*)]$ is decreasing in the confidence level $\alpha$. This completes the proof.

The confidence level $\alpha$ reflects the degree of risk aversion of the newsvendor and the bigger the confidence level $\alpha$ is, the more risk-averse the newsvendor is.

## 4. Numerical Results

In this section, we will give an example to show the results obtained in Section 3 and present some management insights obtained from the numerical results.

**Example 4.1** For the newsvendor model, suppose the market demand $\zeta$ subjects to the uniform distribution $U(0,\ 1000)$. Moreover, the other parameters are given as $p$=8, $c$=5 and $r$=2. For these parameters, let us compute the optimal order quantities $q^*$ and $q_\alpha^*$ above for the newsvendor, and give a sensitivity analysis.

First, let $w$=0.5, $\lambda$=2 and $\alpha$=0.5, we compute the optimal order quantities $q^*$ and $q_\alpha^*$ with different parameters $p$, $c$ and $r$ for the newsvendor, and present a sensitivity analysis in Figure 1, 2 and 3 respectively.

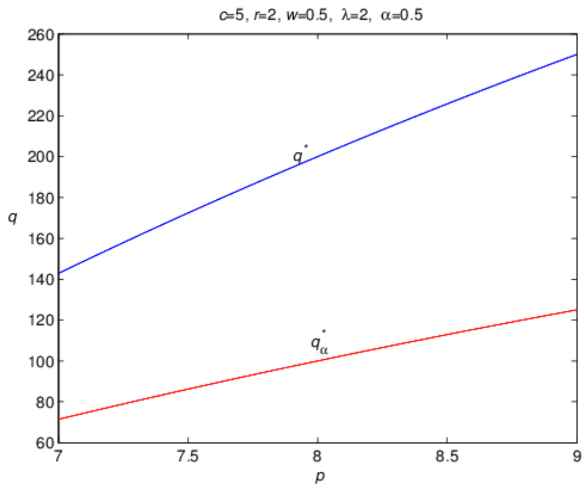


Fig. 1. Optimal order quantities

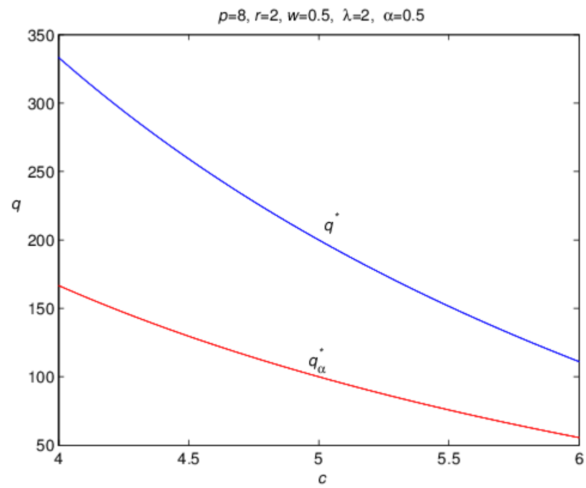


Fig. 2. Optimal order quantities

By Figure 1, it is easily checked that $q^*$ and $q^*_\alpha$ both are increasing in the retail price $p$ and it satisfies $q^*_\alpha < q^*$ for different $p$.

Further, Let $p$=8, $c$=5, $r$=2 and $\alpha$=0.5, we compute the optimal order quantities $q^*$ and $q^*_\alpha$ with different backorder rate $w$ and loss aversion coefficient $\lambda$ for the newsvendor, and present a sensitivity analysis in Figure 4 and 5 respectively.

By Figure 4, the optimal order quantities $q^*$ and $q^*_\alpha$both are decreasing in the backorder rate $w$. Besides, it also satisfies $q^*_\alpha < q^*$ for different backorder rate $w$.

By Figure 5, the optimal order quantities $q^*$ and $q^*_\alpha$ both are decreasing in the loss aversion coefficient$\lambda$. Besides, it also satisfies $q^*_\alpha < q^*$ for different loss aversion coefficient $\lambda$.

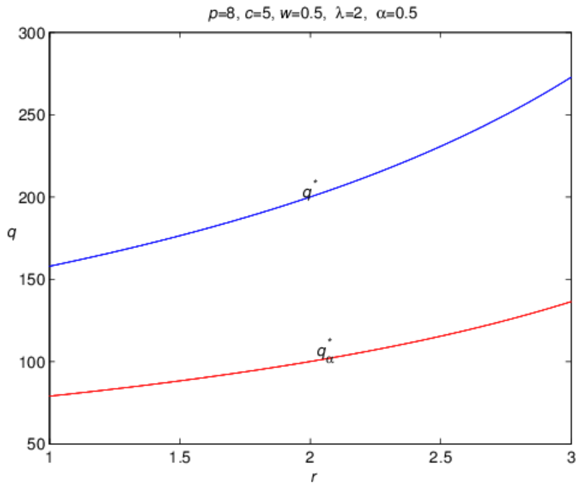


Fig. 3. Optimal order quantities

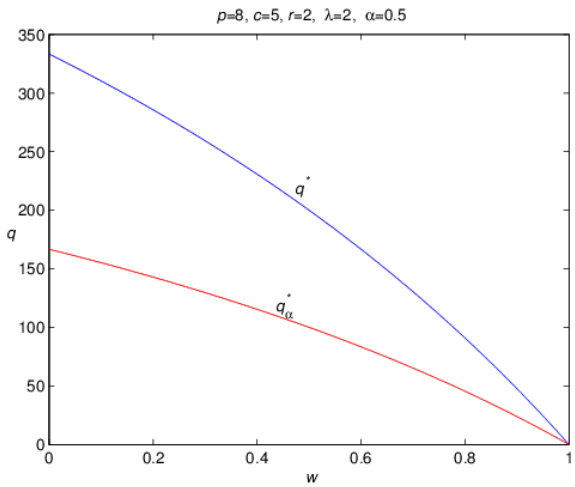


Fig. 4. Optimal order quantities

Finally, Let $p{=}8$, $c{=}5$, $r{=}2$, $w{=}0.5$ and $\lambda{=}2$, we compute the optimal order quantities $q^*$ and $q^*_\alpha$ with different confidence level $\alpha$ for the newsvendor, and present a sensitivity analysis in Figure 6. By Figure 6, the optimal order quantity $q^*(\alpha{=}0)$ stays the same and the optimal order quantity $q^*_\alpha$ is decreasing in the confidence level $\alpha$. Besides, it also satisfies $q^*_\alpha < q^*$ for different confidence level $\alpha$.

To summarize this section, the numerical experiment and sensitivity analysis confirm that the results obtained in the above section are qualitatively robust.

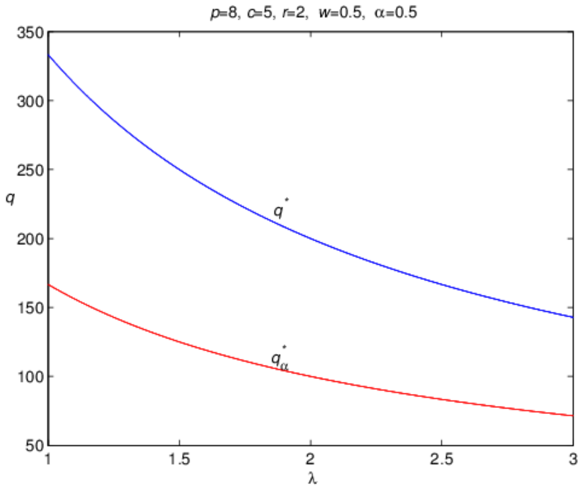


Fig. 5. Optimal order quantities

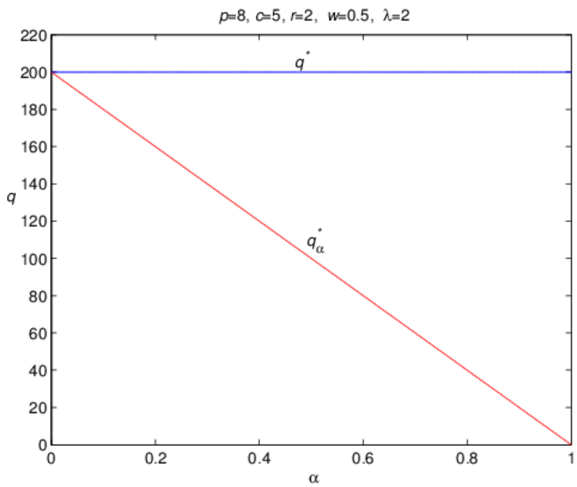


Fig. 6. Optimal order quantities

## 5. Conclusion

In the newsvendor model with a backorder, the losses mainly come from the excess order, and the lost sales can be reduced by reorder. In this case, the newsvendor is more averse to the loss. However, the study about the influence of the loss-averse newsvendor model with a backorder is very few. Therefore, this paper contributes to the study about the optimal order quantity decisions of such a loss-averse newsvendor model with a backorder. Firstly, by introducing the backorder rate $w$ and the loss aversion coefficient$\lambda$, we propose a novel utility function for the loss-averse newsvendor. Then, by maximizing the expected utility, we obtain the optimal order quantity for the risk-neutral newsvendor who is loss-averse. By maximizing the CVaR about utility, we have the optimal order quantity for the risk-averse newsvendor who is loss-averse, too. The methods can help the newsvendor permit a backorder to reduce the risk originating from the fluctuation in the market demand. It is found that, by maximizing the expected utility of the loss-averse newsvendor permit a backorder, the optimal order quantity is smaller than the newsvendor without a backorder. Just as the conclusions in [8], we also found that, in the loss-averse newsvendor model with a backorer, if the loss-averse newsvendor is risk-averse, his/her optimal order quantity to maximize the CVaR about utility is decreasing in the confidence level$\alpha$, and the expected utility under such an optimal order quantity is decreasing in the confidence level$\alpha$as well. This verifies that if the loss-averse newsvendor permit a backorder selects an order quantity to reduce/control the potential risk, he/she will expect a lower utility. Besides, it is shown that if the newsvendor becomes more loss-averse, then he/she will order less products to maximize his/her expected utility or CVaR about utility. Thus, this research also shows how the loss aversion influence the optimal order quantity decisions in the newsvendor model and may present some policies to mitigate the decision bias in the classical newsvendor model.

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